③

Part(2) & (3)

⇒ projectiles.

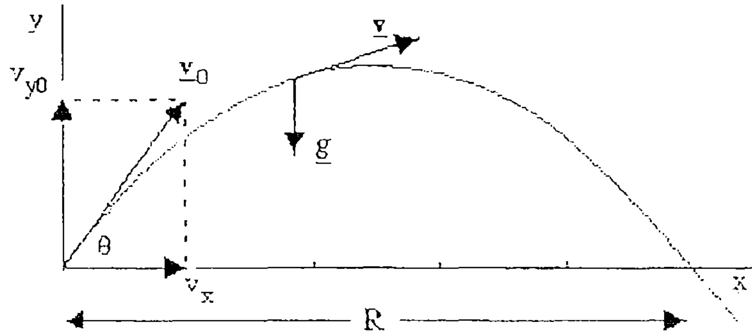
⇒ Normal & Tangent.

جامعة حلوان
Helwan University

Final_Revision

In

Dynamics



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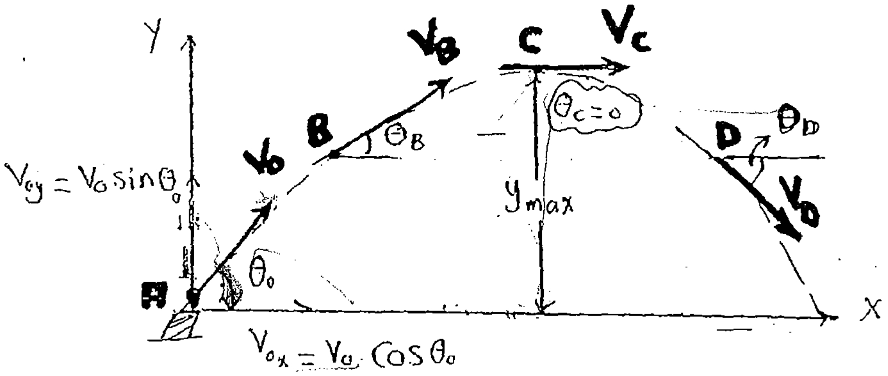
جزء ١ العدل ←

Chapter (1)

Projectile: (حركة المقذوفات)



ملاحظات هامة :

① - دائما السرعة عند اى نقطة على مسار حركة المقذوف تكون مماسية للمسار ومع اتجاه الحركة .

② - إذا كانت السرعة مائلة يجب تحليلها لمركبتين (V_x ، V_y) .

③ - زاوية ميل السرعة الكلية عند أى نقطة هى الزاوية المحصورة بين السرعة الكليه والموازى للأفقى .

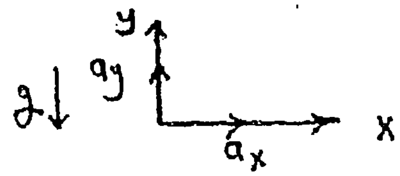
④ - السرعة عند أعلى نقطة على المسار دائما أفقية وبالتالى لا يوجد لها مركبة رأسية ($V_y = 0$) ، وذلك لأن السرعة الكلية موازية لمحور X . ($V = V_x$)

⑤ - حركة المقذوفات تعتبر من أهم تطبيقات العجلة الثابتة ، لذلك نستخدم قوانين العجلة الثابتة .

$$V = V_0 + a t$$
$$S - S_0 = V_0 t + \frac{1}{2} a t^2$$
$$V^2 = V_0^2 + 2a (S - S_0)$$

⑥ - قيمة العجلة الثابتة المؤثرة أثناء حركة المقذوفات هي عجلة الجاذبية

ولذلك لا يوجد مركبة افقية للعجلة في اتجاه x (فقط في اتجاه y)



$a_y = -g$ ، $a_x = 0$

Equations of Motion (معادلات الحركة)

نطبق قوانين العجلة الثابتة في اتجاه x وفي اتجاه y لإيجاد احداثيات اي نقطة على المسار وإيجاد مركبتي السرعة عند نفس النقطة

→ (x, y) للاحداثيات

→ (V_x, V_y) مركبات السرعة

X - Dir

$a_x = \text{zero}$

$V_x = \text{Const}$ ثابتة خلال المسار كله.

$$X - X_0 = V_{0x} \cdot t + \frac{1}{2} \cancel{a_x t^2} \to (0)$$

$$\boxed{X - X_0 = V_{0x} \cdot t} \to ①$$

$$V_x = V_{0x} + \cancel{a_x \cdot t} \to (0)$$

$$\boxed{V_x = V_{0x}} \to ②$$

تستخدم قوانين السرعة في حالة ايجاد مركبات السرعة عند نقطة

Note:

$$\boxed{V_{0x} = V_0 \cdot \cos\theta_0}$$

او السرعة الابتدائية

Y - Dir

$a_y = -g$ → $9.8\ m/s^2$, → $32.2\ ft/s^2$

$$y - y_0 = V_{0y} \cdot t + \frac{1}{2} a_y t^2$$

$$\boxed{y - y_0 = V_{0y} \cdot t - \frac{1}{2} g t^2} \to ③$$

$$V_y = V_{0y} + a_y \cdot t$$

$$\boxed{V_y = V_{0y} - g t} \to ④$$

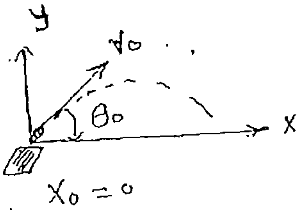
$$V_y^2 = V_{0y}^2 + 2a_y \cdot (y - y_0)$$

$$\boxed{V_y^2 = V_{0y}^2 - 2g(y - y_0)} \to ⑤$$

Note:

$$\boxed{V_{0y} = V_0 \cdot \sin\theta_0}$$

ملاحظات هامة على معادلات الحركة :



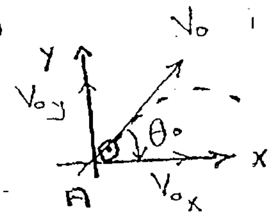
$x_0 = 0$
$y_0 = 0$

① - يُفضل وضع محاور الحركة عند نقطة القذف

② - نستخدم المعادلات الخمسة حيث المعادلة رقم ① لقيمة الـ X ، والمعادلة رقم ③ قيمة الـ y ، والمعادلات ② ، ④ ، ⑤ لمركبتي السرعة $[V_x , V_y]$.

نقاط هامة على مسار أى مقذوف :

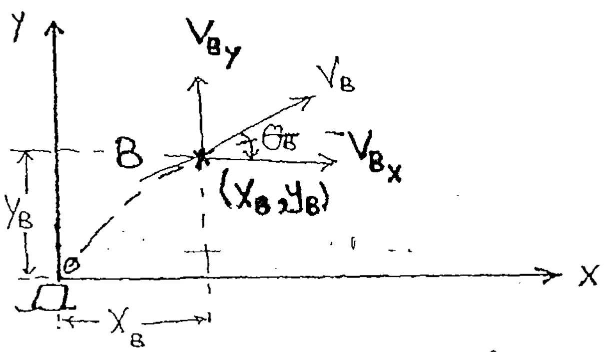
Point (A) : نقطة القذف



ونحصل منها دائما على قيم السرعة الابتدائية ← V_0
(initial velocity = muzzle velocity)

بالاضافة للزاوية بين V_0 والمحور الافقى ← θ_0

Point (B) : اى نقطة خلال رحلة الصعود



لمعلومية أى معلومتين عند النقطة B (X_B , Y_B)
أو مركبتى السرعة (V_{Bx} , V_{By}) نستخدم القوانين السابقة الخمسة .

EX: $X_B - \cancel{X_0}^{\,0} = V_{ox} \cdot t_B \rightarrow ①$ | $y_B - \cancel{y_0}^{\,0} = V_{oy} \cdot t_B - \frac{1}{2} g t_B^2 \rightarrow ③$

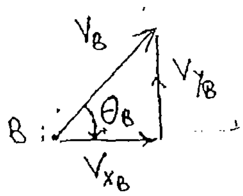
نعوض فى المعادلات لإيجاد المجاهيل المطلوبة .

نحسب مركبات السرعة V_x ، V_y من المعادلات رقم (4) ، (2) .

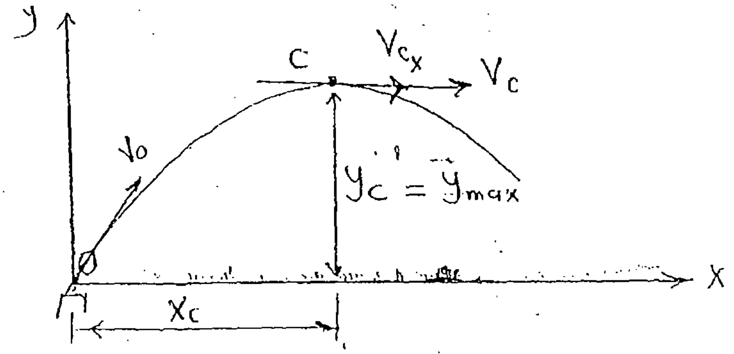
ثم نستخدمهم لإيجاد السرعة الكلية عند النقطة المطلوبة ولتكن B

(بالمقدار) $V_B = \sqrt{V_{x_B}^2 + V_{y_B}^2}$.

(الإتجاه) $\theta_B = \tan^{-1} \frac{V_{y_B}}{V_{x_B}}$.



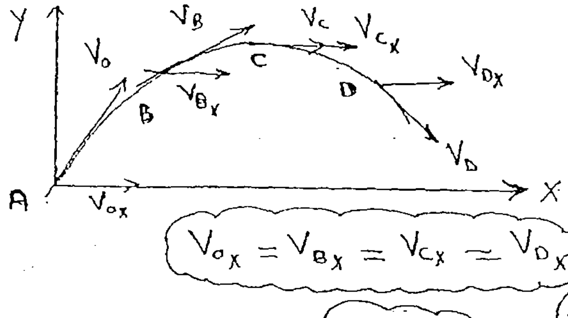
At Point (C) : (أعلى نقطة على المسار)



نلاحظ ان السرعة الكليه عند النقطة (C) افقية و بالتالي مركبتها الرأسية تساوي صفر . $V_{y_C} = 0$ ، ايضا لا يوجد زاوية للميل على الأفقي $\theta_C = 0$

$V_C = V_{C_x} = Const$

تذكر : السرعة الافقية دائما ثابتة طول المسار كله عند أي نقطة



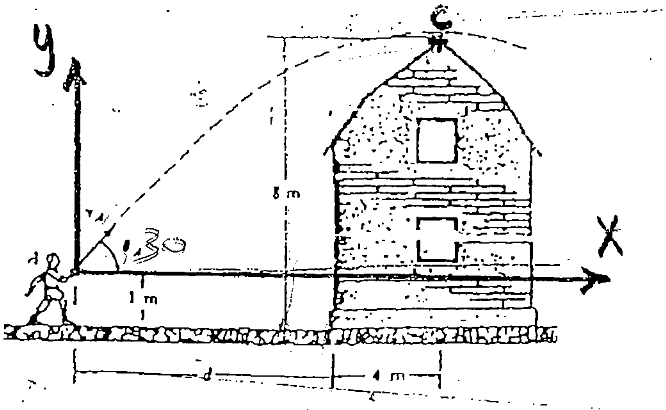
$V_{0x} = V_{Bx} = V_{Cx} = V_{Dx}$

$\because a_x = 0 \Rightarrow V_x = Const$

لاحظ : نستخدم عند هذه النقطة دائما المعادلة رقم (5)

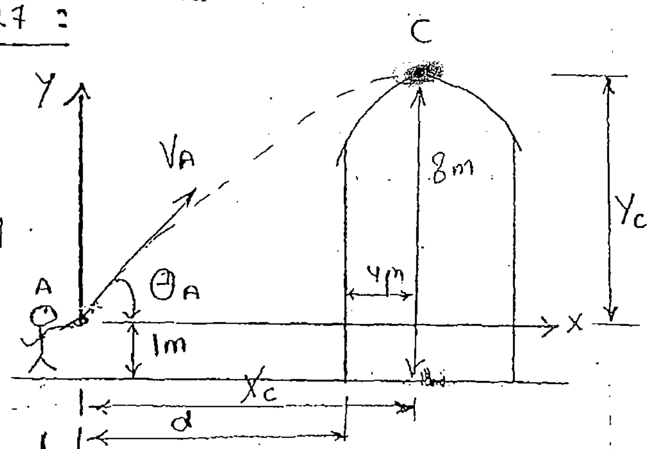
$V_{y_C}^2 = V_{0y}^2 - 2g(y_C - y_0) \Rightarrow V_y = 0$

The boy at A attempts to throw a ball over the roof of a country house with an angle $\theta_A=30°$. Determine the initial velocity v_A at which the ball must be thrown so that it just clears the peak at C. Also, find the distance d where he should stand to throw the ball.



Prob ① May 2011 Page 27 :

- $\theta_A = 30°$
- at Point C $\Rightarrow$ (Peak)
- $V_A = ??$? $d = ??$



Sol;

at Point (C):

at Max hieght

$(V_y = 0)$, $X_c = 4+d$.

$y_c = 8-1 = 7\,m$.

$V_{ox} = V_A \cos 30 = \sqrt{\frac{3}{2}}\, V_A$.

$V_{oy} = V_A \sin 30 = 0.5\, V_A$.

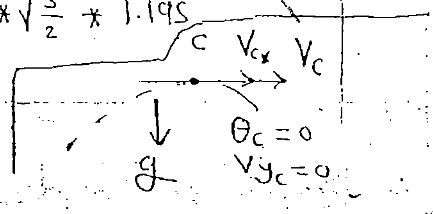
$zero = V_{oy}^2 - 2g(y_c - \underbrace{y_o}_{0})$

$0 = (0.5 V_A)^2 - 2 * 9.8 (7) \Rightarrow \boxed{V_A = 23.44\, m/s}$

$V_y = V_{oy} - g t_c \Rightarrow 0 = (23.44 * 0.5) - 9.8\, t_c \Rightarrow \boxed{t_c = 1.195\, sec}$

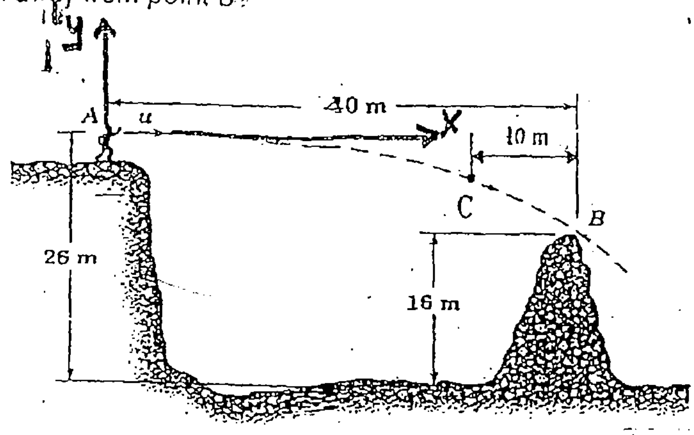
$X_c - \underbrace{X_o}_{0} = V_{ox} . t_c \Rightarrow d + 4 = 23.44 * \sqrt{\frac{3}{2}} * 1.195$

$\boxed{d = 20.25\, m}$



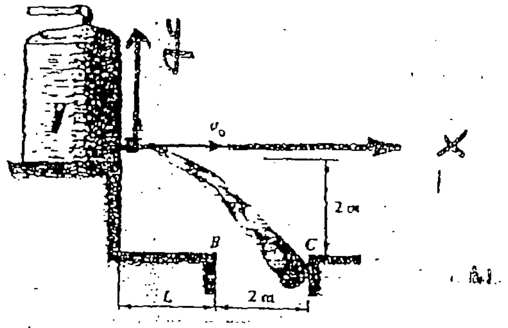
(1)

With what minimum horizontal velocity 'u' can a boy throw a rock at A and have it just clear the obstruction at B? What is the total velocity of the rock at the point C which is 10 m away from point B?

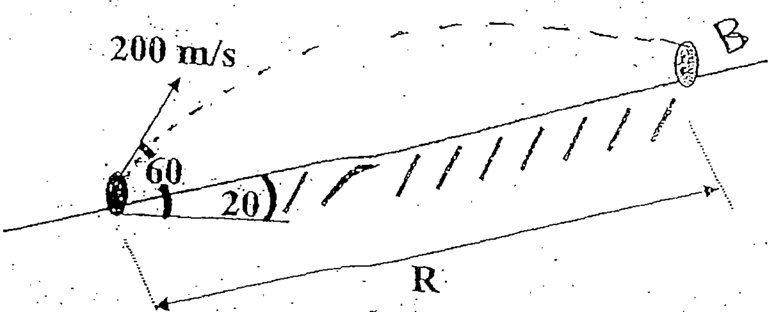


(2)

Water is discharged at A from a pressure tank with a horizontal velocity v_o as shown. if L=4 m, determine the range of values of v_o for which the water will enter the drainage opening BC. Assume the water enter at the middle of the drainage, find the total water velocity and draw its components at the entering,



(3) A projectile is launched from point A with the initial conditions shown in the figure. Compute the range R as measured up the incline.



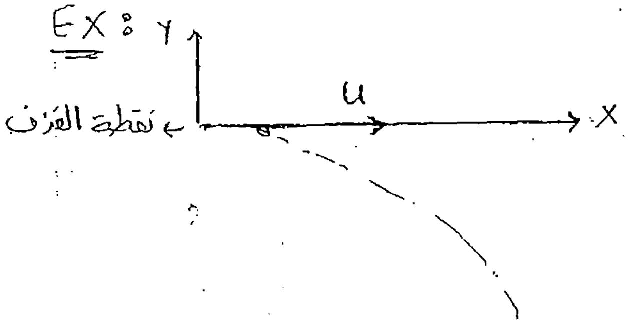
2^{nd} Idea : (الفكرة الثانية)

← إذا كانت السرعة الإبتدائية أفقية يذكر في المسألة

horizental Velocity .

او يضع سهم افقي عند نقطة القذف

EX :



$V_o = u$
$\theta_o = 0$

$$\boxed{\begin{array}{l} V_{ox} = V_o = u \\ V_{oy} = \text{zero} \end{array}}$$

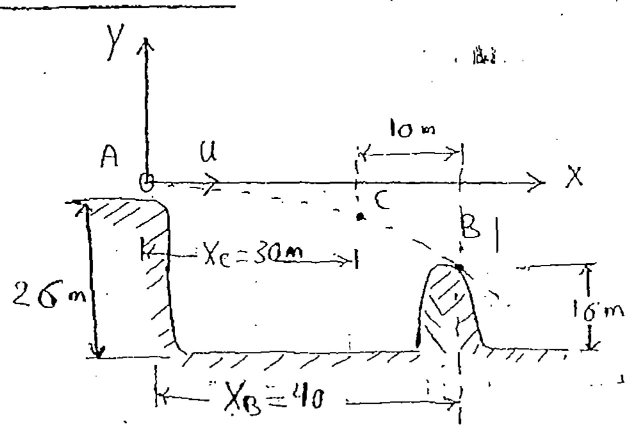
Prob (2) : May 2004 Page (12) :

$u = ??$

V at Point $C = ??$

Sol

$$\boxed{\begin{array}{l} V_{ox} = u \\ V_{oy} = 0 \end{array}}$$



At Point (B) : (موجود عندها معلومات اكثر)

$$X_B = 40\,m \;,\; y_B = -(26-16) = -10\,m$$

$$X_B = V_{ox} \cdot t_B \Rightarrow 40 = u \cdot t_B \Rightarrow \boxed{t_B = \frac{40}{u}}$$

$$y_B = \cancel{V_{oy} t_B} - \frac{1}{2} g t_B^2$$

$$-10 = -\frac{1}{2} \times 9.8 \left(\frac{40}{u}\right)^2 \Rightarrow \boxed{u = 28\ m/s}$$

At Point (C) : (مطلوب قيمة السرعة الكليه)

لذلك نحسب مركبتيها $[V_{x_c}, V_{y_c}]$ ، لكن يجب ايجاد t_c أولا

∴ نستخدم المعلومة الموجودة $(X_c = 30)$

$$X_c = V_{ox} \cdot t_c \Rightarrow 30 = 28 * t_c$$

$$\boxed{t_c = \frac{30}{28} = 1.07 \text{ sec}}$$

$$V_{y_c} = \cancel{V_{oy}} - g \cdot t_c \Rightarrow V_{y_c} = -9.8 * 1.07$$

$$\boxed{V_{y_c} = -10.486 \text{ m/s}}$$

$$\boxed{V_{x_c} = u = 28 \text{ m/s}}$$

Total Velocity at Point (C):

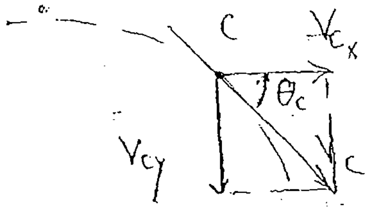
$$V_c = \sqrt{V_{x_c}^2 + V_{y_c}^2}$$

$$V_c = \sqrt{(-10.486)^2 + (28)^2} = 29.899 \text{ m/s}$$

المقدار $\boxed{V_c \simeq 30 \text{ m/s}}$

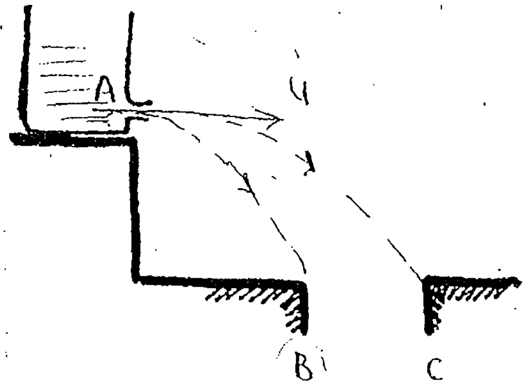
الاتجاه $\theta_c = \tan^{-1} \frac{V_{y_c}}{V_{x_c}} = \tan^{-1} \frac{-10.486}{28}$

$$\boxed{\theta_c = 20^\circ}$$



Idea ③ : (الفكرة الثالثة)

⇐ إذا طلب مدى السرعات الابتدائية التي تسمح بدخول المقذوف خلال الفتحة .

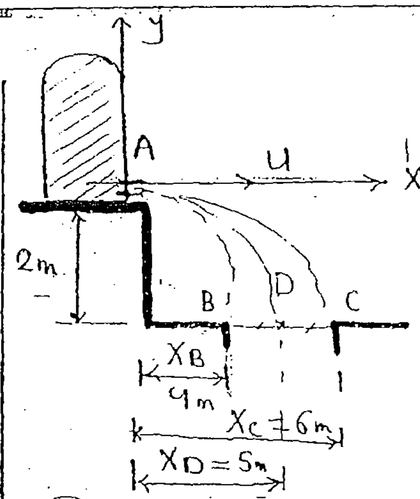


⇐ الرسمة عبارة عن منحنيين :

① المنحنى الاول يبدأ من النقطة A ويصل للنقطة B . وتعتبر قيمة السرعة الابتدائية له هي القيمة الأقل (U_{min}) .

② المنحنى الثاني يبدأ من النقطة A (نقطة القذف) ويصل للنقطة C والسرعة الابتدائية له هي اعلى قيمة (U_{max}) .

Prob ③



Req : ① Range of u .
② V_{tot} at point D and sketch .

at point (B) to get (U_{min}):

$X_B = 4\,m$, $y_B = -2\,m$

$X_B = V_{ox} \cdot t_B \Rightarrow 4 = U_{min} * t_B$

$$\boxed{t_B = \frac{4}{U_{min}}}$$

$V_{ox} = u$
$V_{oy} = 0$ → السرعة الابتدائية أفقية

$$y_B = \cancel{V_{oy}}\, t_B - \frac{1}{2} g t_B^2 \Rightarrow \cancel{-}2 = \cancel{-}\frac{1}{2} * 9.8 \left(\frac{4}{U_{min}}\right)^2$$

$V_{oy} = 0$

$$\boxed{U_{min} = 6.26 \ m/s}$$

<u>at Point (c) → to get (U_{max}):</u>

$X_c = 6m$, $Y_c = -2$

$$X_c = V_{ox} \cdot t_c \Rightarrow 6 = U_{max} * t_c \Rightarrow \boxed{t_c = \frac{6}{U_{max}}}$$

$$Y_c = \cancel{V_{oy}}\, t_c - \frac{1}{2} g t_c^2 \Rightarrow -2 = -\frac{1}{2} \times 9.8 \left(\frac{6}{U_{max}}\right)^2$$

$V_{oy} = 0$

$$\boxed{U_{max} = 9.39 \ m/s}$$

⊛ <u>total Velocity at middle of drianage (at Point D)</u>

$$V_{xD} = U_o = \frac{U_{max} + U_{min}}{2} \Rightarrow U_o = \frac{6.26 + 9.39}{2}$$

$$\boxed{U_o = 7.825 \ m/s}$$

<u>to get (t_D):</u> (X_D معلومة)

$$X_D = 5 \Rightarrow 5 = V_{ox} \cdot t_D \Rightarrow 5 = 7.825 * t_D$$

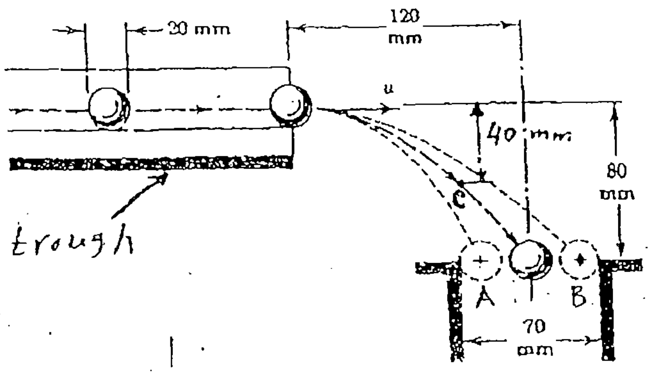
$$t_D = \frac{5}{7.825} \Rightarrow \boxed{t_D = 0.6389 \sec}$$

$$V_{y_D} = \cancel{V_{oy}} - g\, t_D \Rightarrow V_{y_D} = -9.8 * 0.6389$$

$V_{oy} = 0$

$$\boxed{V_{y_D} = -6.2619 \ m/s}$$

Bearing balls leave the horizontal trough with a velocity of magnitude u and fall through the 70 mm diameter hole as shown. Calculate the permissible range of u which will enable the balls to enter the hole. Take the dotted positions to represent the limiting conditions. If the balls enter in the middle of the hole, calculate and draw the velocity components at the point **c**.

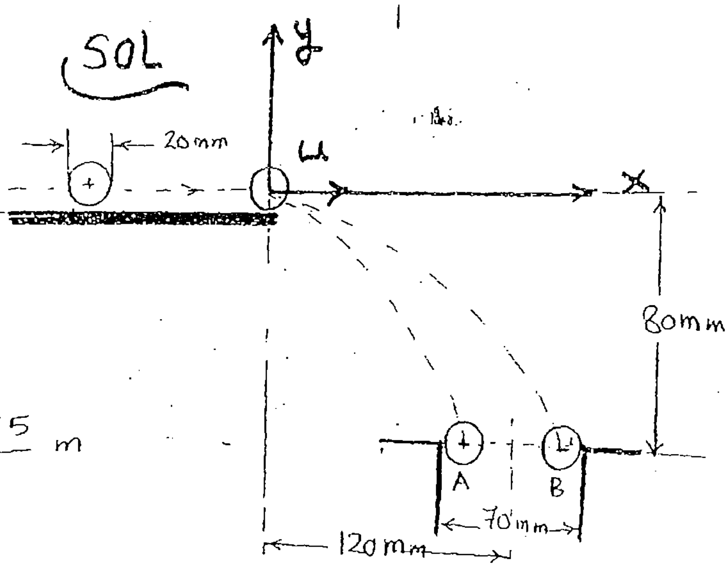


SOL

لاحظ :

نأخذ الإحداثيات هنا من

مركز الكرة لأنها لها قطر !



At Point A :

$u = u_{min}$, $x = \frac{120 - 25}{1000}$ m

$x = 0.095$ m

$y = -0.08$

$x = V_0 t \cancel{\cos\theta}_{\to 1} = u_{min} * t$ السرعة الابتدائية أفقية ⇐ $(\theta = 0)$.

$$t = \frac{x}{u_{min}} = \frac{0.095}{u_{min}}$$

$$y = V_0 \cancel{t \sin\theta}_{\to 0} - \frac{1}{2} g t^2 \Rightarrow -0.08 = -\frac{1}{2} * 9.81 \left(\frac{0.095}{u_{min}}\right)^2$$

$$\boxed{u_{min} = 0.74 \text{ m/s}}$$

At Point B: $u = u_{max}$, $y = +0.08$ m

$x = 0.145$ m

$$x = V_o t \underbrace{\cancel{\cos}\,\theta}_{1} \Rightarrow 0.145 = u_{max} * t$$

$$t = \frac{0.145}{u_{max}}$$

$$y = V_o t \underbrace{\cancel{\sin}\,\theta}_{zero} - \frac{1}{2} g t^2 \Rightarrow -0.08 = -\frac{1}{2} * 9.81 * \left(\frac{0.145}{u_{max}}\right)^2$$

$$\boxed{u_{max} = 1.135 \text{ m/s}}$$

$$\boxed{0.74 \leqslant u \leqslant 1.135}$$ → range of intial velocity.

At Point C: $y = -0.04$, $V_x = u = (u_{max} + u_{min})$

$$\boxed{V_x = 0.9375 \text{ m/s}}$$

$$y = V_o t \underbrace{\cancel{\sin}\,\theta}_{zero} - \frac{1}{2} g t^2 \Rightarrow -0.04 = -\frac{1}{2} \times 9.81$$

$$\boxed{t = 0.09 \text{ sec}}$$

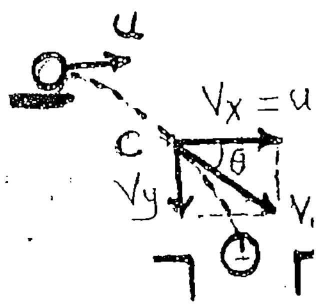
$$V_y = V_o \underbrace{\cancel{\sin}\,\theta}_{0} - g t \Rightarrow V_y = -9.81(0.09)$$

$$\boxed{V_y = -0.8829 \text{ m/s}}$$

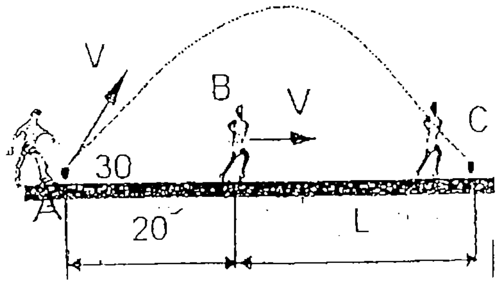
$$\theta_c = \tan^{-1} \frac{V_y}{V_x} = \tan^{-1} \frac{-0.8829}{0.9375} = -43.28$$

$$V_c = \sqrt{V_x^2 + V_y^2} = \sqrt{0.9375^2 + 0.8829^2}$$

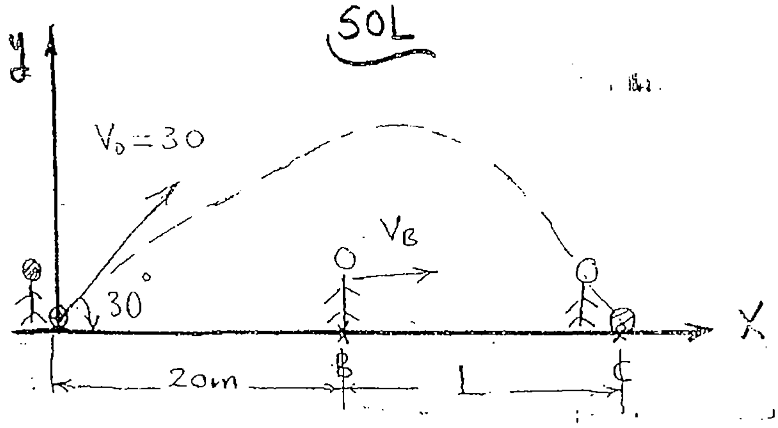
$$\boxed{V_c = 1.2878 \text{ m/s}}$$



At a given instant a football player at point A throw a football with a velocity Vo = 30 m/s as shown. At this moment, another player at B was 20 m a way from Point A. What is the constant speed at which the player at B must run so that he can catch the ball at point C?



SOL



= المسألة يوجد بها نوعين من الحركة
- حركة على خط مستقيم بسرعة ثابتة (اللاعب)
- حركة على منحنى (مقذوفات) (الكرة)

= الشئ المشترك بين اللاعب والكرة هو تساوى الوقت لهما.

$$\boxed{t_{Ball} = t_{player}}$$

for Ball :

at Point C : $X = 20 + L$ & $y = 0$

$X = V_o t \cos\theta \Rightarrow 20 + L = 30 t \cos 30$

$\boxed{L = 30t \cos 30 - 20} \rightarrow$ (I)

$y = V_o t \sin\theta - \frac{1}{2} g t^2 \Rightarrow 0 = 30 t \sin 30 - \frac{1}{2} \times 9.81 t^2$

$4.905 t^2 - 15 t = 0$

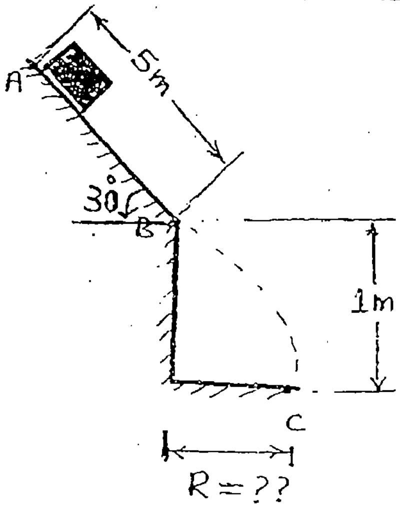
$t (4.905 t - 15) = 0$

$t = 0$ rejected | $\boxed{t = 3.058 \text{ sec}}$ sub in (I)

$L = 30 (3.058) \cos 30 - 20$

$\boxed{L = 59.45 \text{ m}}$

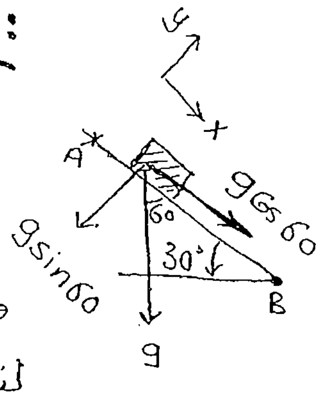
A particle start at A from rest & find R.



SoL

From A to B:

⇐ الجسم يتحرك تحت تأثير عجلة الجاذبية لأسفل ولكنه يميل على الأفقي ، لذلك نأخذ مركبة العجلة في إتجاه الحركة (a_x) .

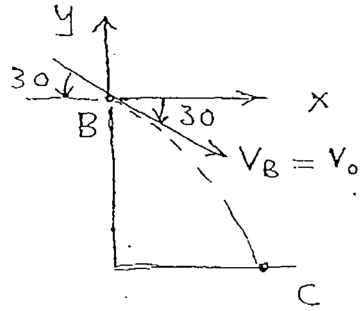


$$a_x = g\cos 60$$
$$a_x = 9.8(\cos 60) = 4.9 \text{ m/s}$$
$$\boxed{a_x = const = 4.9 \text{ m/s}^2}$$

$$V_B^2 = V_A^2 + 2(a_x)\, S_{A \to B}$$
$$V_B^2 = \text{Zero} + 2(4.9)(5) \Rightarrow \boxed{V_B = 7 \text{ m/s}}$$

From B to C:

⇐ هذا الجسم يتحرك كمقذوف بسرعة إبتدائية تساوى السرعة عند النقطة (B) .



At point C: $y = -1$ & $X = R$

$$y = X\tan\theta_i - \frac{gX^2}{2V_o^2}(1 + \tan^2\theta_i)$$

$$-1 = R\tan(-30) - \frac{9.8R^2}{2(7)^2}(1 + \tan^2(30))$$

$$\boxed{R = \checkmark\checkmark \text{ m}}$$

Note:

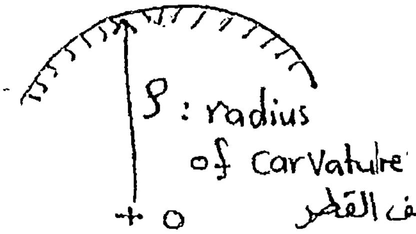
$\theta_i = -30$
$= 330$

لأنها في الربع الرابع .

Normal & Tangential Coordinates (n,t)

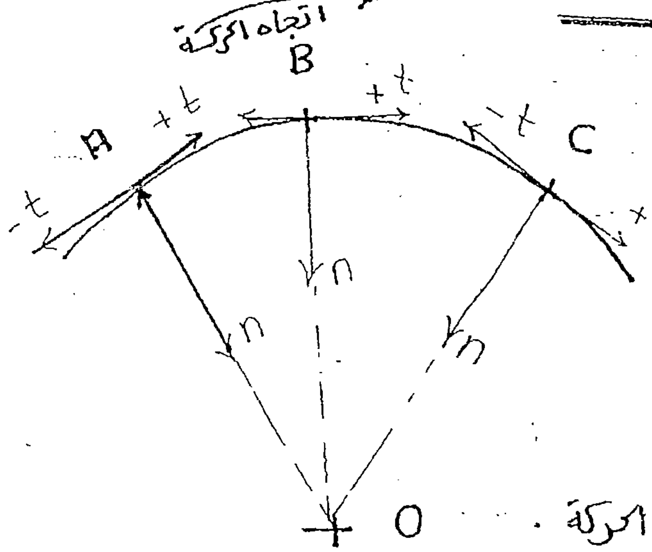
المحور العمودى
والمماس

نستخدم هذه المحاور لوصف حركة اى جسم على مسار دائرى
له نصف قطر معين .



← لو ضع المحاور هنا يجب أن نعلم ان المحاور توضع عند كل نقطة
ولا يوجد لها نقطة أصل معينة ولذلك تسمى احيانا المحاور المتحركة .

← رسم المحاور (n,t) :

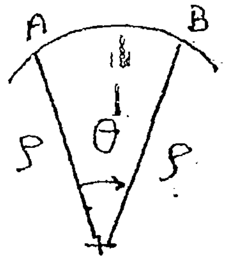


لاحظ :

$\boxed{1}$ **t** : المحور المماس
ويكون مماس للمسار الدائرى عند
النقطة ، واتجاهه الموجب مع
اتجاه الحركة والسالب عكس اتجاه الحركة .

$\boxed{2}$ **n** : المحور العمودى ويكون عمودى على محور t فى نفس اتجاه المركز
ودائما اتجاهه داخل ناحيه مركز الدوران (لا يوجد له سالب) .

$\boxed{1}$ Displacement (ΔS) الإزاحة



$$\underset{A\to B}{S} = \rho * \theta$$

$$\theta \Rightarrow degree * \frac{\pi}{180} = \text{٧٧ rad}$$

2 Velocity: السرعة

a $V = Const = Max\ speed$

$$\boxed{V = \frac{S}{t}}$$

اذا كانت السرعة الثابتة نستخدم قوانين السرعة الثابتة

b $V = f(t)$ متغيرة مع الزمن

نستخدم العلاقات الآتية لتحديد الازاحة والعجلة

$$\frac{dv}{dt} = a_t \quad , \quad \frac{ds}{dt} = V$$

c V (change with uniform rate) السرعة تتغير بمعدل منتظم

$\therefore$ ($a_t = Const$) or (uniform acc.)

ونستخدم هنا قوانين العجلة الثابتة

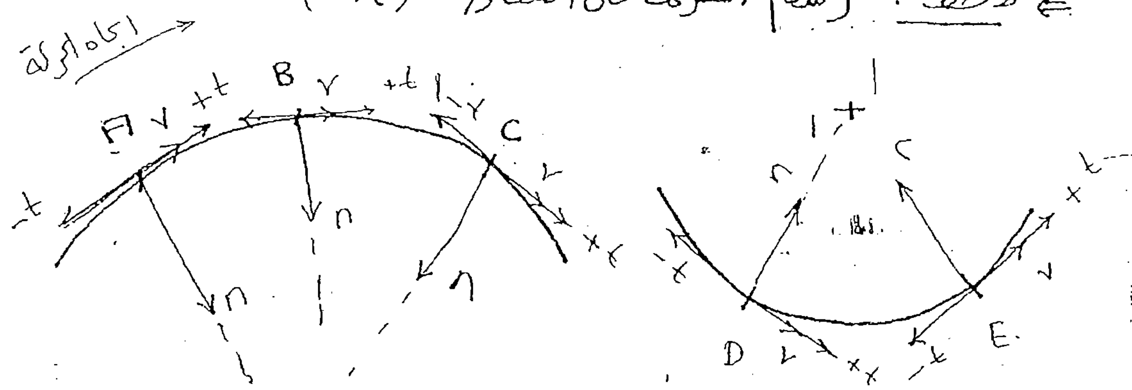
$$\boxed{\begin{aligned} V &= V_o + a t \\ S - S_o &= V_o t + \frac{1}{2} a t^2 \\ V^2 &= V_o^2 + 2a(S - S_o) \end{aligned}}$$

لاحظ: السرعة الكلية عند اى نقطة اتجاهها موازى لمحور t لذلك لا يتم تحليلها.

Note: V is decrease uniform $\rightarrow a_t = -$ Value

increase uniform $\rightarrow a_t = +$ Value.

← لاحظ: رسم السرعة على المحاور (n, t)

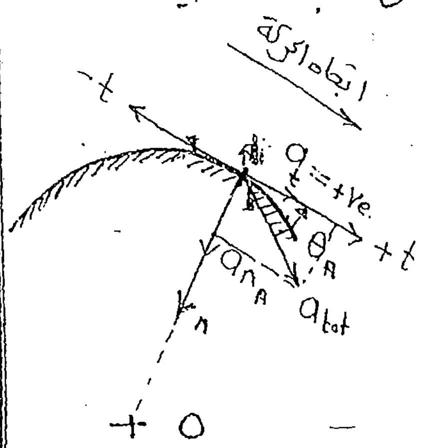


← دائما اتجاه السرعة (تزيد او تقل) فى اتجاه المماس مع اتجاه الحركة وبالتالى مع t^+

← السرعة هنا فى محاور (n, t) ليس لها الا مركبة واحدة هى السرعة الكلية موازية لمحور t^+.

3 acceleration: (العجلة)

العجله في محاور (n,t) لها مركبتان وذلك لان العجلة الكليه تميل على المحور المماس t .



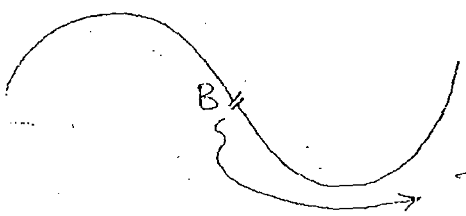
لاحظ :

العجلة لهـــا مركبتان :

1 المركبة العمودية : a_n

ودائما اتجاهها داخل للمركز مع محور n . قيمتها نحسبها من العلاقة

$$\boxed{a_n)_A = \frac{V_A^2}{\rho_1}}$$



Inflection point ⇒ نقطة الانقلاب

بين المسار الدائريين وتكون قيمة الـ ρ عندها $= \infty$.

$$\boxed{\therefore \underbrace{a_n)}_{\text{Inflection Point}} = \frac{V_B^2}{\rho_B} = \frac{V_B^2}{\infty} = \text{Zero}}$$

2 المركبة المماسية : a_t

بما انها منطبقة على السرعة في الاتجاه واتجاهها على نفس المحور فإنها مرتبطة بنوع السرعة كما وضحنا سابقا في السرعة .

a $V = \text{Const}$ or Max speed $\Longrightarrow a_t = \text{Zero}$.

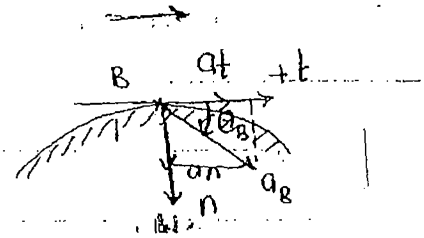
b $V = F(t) \Longrightarrow a = \frac{dv}{dt}$ or $a = F(t)$

c V (change with uniform rate) $\Rightarrow a_t = \text{Const} = \text{uniform}$ نستخدم قوانين العجلة الثابتة .

* total acceleration :

(المقدار) $a_B = \sqrt{a_{n_B}^2 + a_{t_B}^2}$

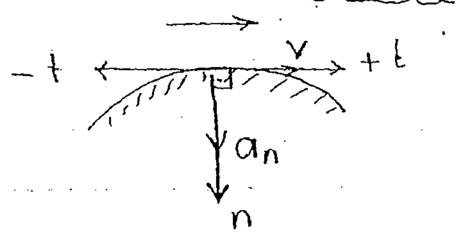
(الإتجاه) $\theta_B = \tan^{-1} \frac{a_{n_B}}{a_{t_B}}$



لاحظ :

بالنسبة لرسم مركبات العجلة ← a_t ، a_n

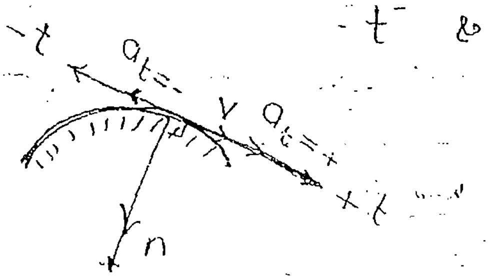
① رسم ال (a_n) :



لا يوجد هنا إلا مركبة موجبة دائما اتجاهها داخل للمركز .

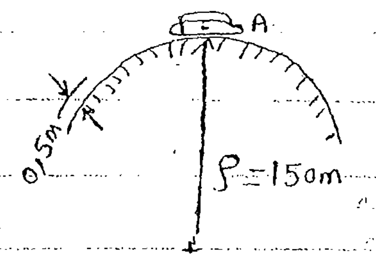
② رسم الـ (a_t) :

لابد قبل رسم ال a_t معرفة اذا كانت تزايديه او تناقصية لان التزايديه مع t^+ والتناقصية مع t^-

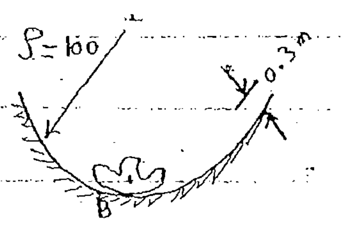


ملاحظة هامة :

إذا أعطى الجسم أبعاد فإن نصف القطر يقاس من مركز الجسم حتى مركز الدوران .



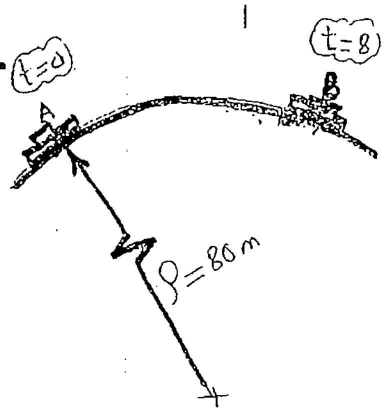
$\rho_A = 150 + 0.5m$



$\rho_B = 100 - 0.3m$

A test car starts from rest on a horizontal circular track of 80 m radius and increases its speed at a uniform rate to each 100 km/h in 10 seconds. Determine the magnitude of the acceleration of the total acceleration of the car 8 seconds after the start. *(Ans. a=6.77 m/s²)*

SOL



$a_t = \text{Const}$

$a_t = 100 \text{ km/h in 10 sec}$

$$a_t = \frac{\cancel{100}^{10} \times \frac{5}{18}}{\cancel{10}}$$

$a_t = 2.78 \text{ m/s}^2$

$V_A = \text{Zero}$ (Start from rest)

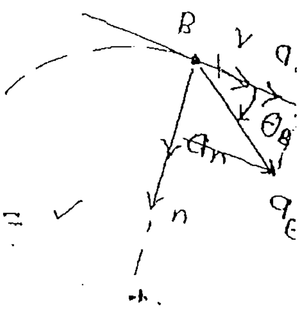
$V_B = V_A + a_t t \Rightarrow V_B = 0 + 2.78(8)$

$$\boxed{V_B = 22.22 \text{ m/s}}$$

$$a_n = \frac{V_B^2}{\rho} = \frac{(22.22)^2}{80} \Rightarrow \boxed{a_n = 6.1728 \text{ m/s}^2}$$

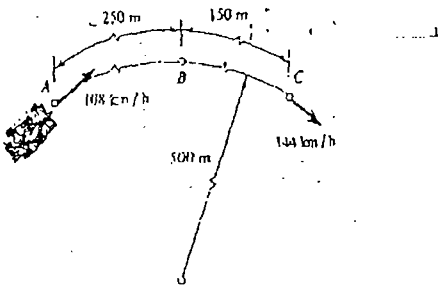
$$a_B = \sqrt{a_n^2 + a_t^2} = \sqrt{(6.17)^2 + (2.78)^2}$$

$$\boxed{a = 6.769 \text{ m/s}^2}$$



$$\theta_B = \tan^{-1}\frac{a_{n_B}}{a_{t_B}} = \tan^{-1}\frac{6.173}{2.78} = ✓$$

A racing car travels along the curve ABC of radius 500 m as shown. The speed of the car increased at a constant rate from 108 km/h at A to 144 km/h at C. Determine the magnitude of the total acceleration of the car when it passes through the point B. Draw the acceleration components at C.



Sol

$$V_A = 108 \times \frac{5}{18} = 30\ m/s \qquad V_C = 144 \times \frac{5}{18} = 40\ m/s$$

the speed is Car increased at a Const rate

يعنى ذلك ان العجلة المماسية ($a_t = Const$) وتزايدية (قيمتها موجبة)

→ to get (a_t):

$$V_C^2 = V_A^2 + 2a_t \cdot (\Delta S_{A\to C})$$

$$40^2 = 30^2 + 2a_t(400)$$

$$\boxed{a_t = +0.875\ m/s^2}$$ → قيمتها ثابتة عند جميع نقط المسار الدائرى

at Point (B):

يجب إيجاد قيمة السرعة V_B أولا للتعويض بها فى قانون a_{n_B}.

$$V_B^2 = V_A^2 + 2a_t(\Delta S_{A\to B})$$

$$V_B = \sqrt{30^2 + 2(0.875)(250)}$$

$$\boxed{V_B = 36.57\ m/s}$$

$$a_{n_B} = \frac{V_B^2}{\rho_B} = \frac{(36.57)^2}{500}$$

$$\boxed{a_{n_B} = 2.675\ m/s^2}$$

$$\boxed{a_t)_B = 0.875\ m/s^2}$$

$$a_t)_B = a_t)_C = a_t)_A = Const$$

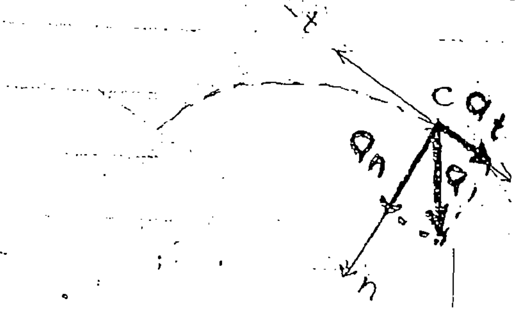
* total acc at Point (B):

$$a_B = \sqrt{a_{n_B}^2 + a_{t_B}^2} = \sqrt{2.675^2 + 0.8}$$

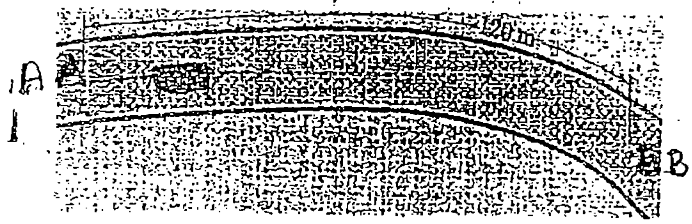
$$\boxed{a_B = 2.8\ m/s^2}$$

$$\theta_B = Tan^{-1}\frac{a_n}{a_t} = Tan^{-1}\frac{2.675}{0.875}$$

* Drawing the acc. Components at (C.



A car travels along a level curved road with a speed that increasing at the constant rate of 0.6 m/s each second. The speed of the car as it passes point A is 16 m/s. Calculate the magnitude of the total acceleration of the car as it passes point B which is 120 m along the road from A. The radius of curvature of the road at B is 60 m. Assuming constant radius of curvature, calculate the time to reach a total acceleration 8 m/s^2. Draw the acceleration components at point B.



SOL

$a_t = Const$ (خلال المسار كله)

$a_t = 0.6$ m/s each second $= 0.6 \frac{m}{s \times s}$

$\boxed{a_t = 0.6\ m/s^2}$ → عجلة تزايدية (في اتجاه الحركة)

For point B:

$$V_B^2 = V_A^2 + 2a_t(S_{A\to B})$$

$$V_B^2 = 16^2 + 2(0.6)(120)$$

$$\boxed{V_B = 20\ m/s}$$

$$a_{n_B} = \frac{V_B^2}{\rho_B} = \frac{(20)^2}{60} \Rightarrow \boxed{a_{n_B} = 6.67\ m/s^2}$$

$$a_B = \sqrt{a_{n_B}^2 + a_{t_B}^2} = \sqrt{(6.67)^2 + (0.6)^2}$$

$$\boxed{a_B = 6.69\ m/s^2}$$

For time at $a = 8\ m/s^2$:

$$a = \sqrt{a_n^2 + a_t^2} \Rightarrow 8 = \sqrt{a_n^2 + a_t^2}$$

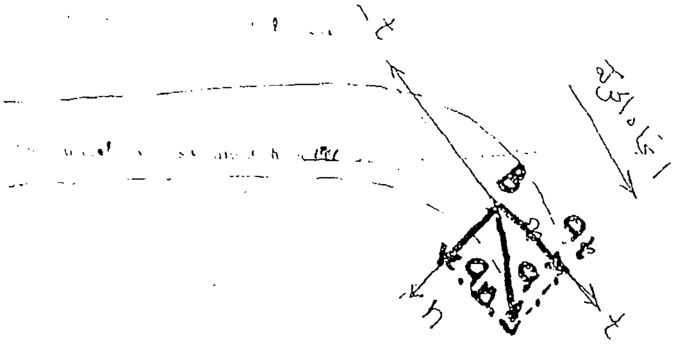
$$8^2 = a_n^2 + a_t^2 \Rightarrow a_n = \sqrt{8^2 - a_t^2}$$

$$a_n = \sqrt{64 - (0.6)^2} \Rightarrow \boxed{a_n = 7.977\ m/s^2}$$

$$a_n = \frac{V^2}{\rho} = \frac{V^2}{60} = 7.997 \Rightarrow \boxed{V = 21.87\ m/s}$$

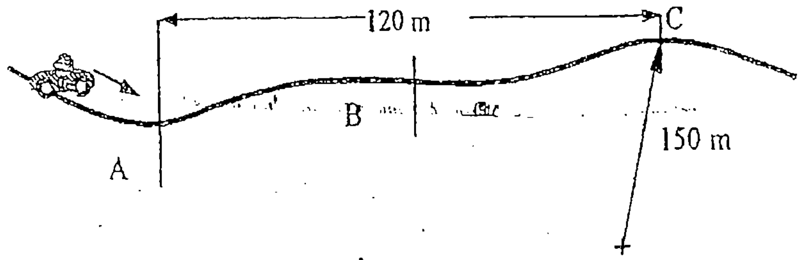
$$V = V_A + a_t t$$

$$21.88 = 16 + 0.6t \Rightarrow \boxed{t = 9.797\ sec}$$



* Components of acc. at Point B:

To anticipate the dip and hump in the road, the driver of a car applies the brake to produce a uniform deceleration. The car speed is 100 km/h at the bottom (point A) of the dip and 50 km/h at the hump (point C), which is 120 m along the road from A. If the total acceleration of the car at A is limited for 3 m/s^2 and the radius of curvature of the hump at C is 150 m, calculate the radius of curvature ρ at A and the total acceleration at the inflection point B and the point C. Draw the acceleration vectors at each point.



SoL

→ Uniform deceleration. ($a_t = const$)

$$V_c^2 = V_A^2 + 2a_t S_{A\to C}$$

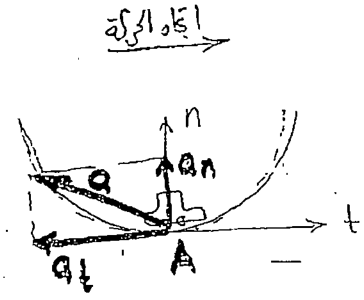
$$\left(50\times\frac{5}{18}\right)^2 = \left(100\times\frac{5}{18}\right)^2 + 2a_t(120)$$

$$\boxed{a_t = -2.41\ m/s^2}$$

→ at point A:

$$a_A = \sqrt{a_t^2 + a_n^2} \Rightarrow 3 = \sqrt{(2.41)^2 + \left(\frac{V_A^2}{\rho_A}\right)^2} \Rightarrow 9 = (2.41)^2 + \left(\frac{V_A^2}{\rho_A}\right)^2$$

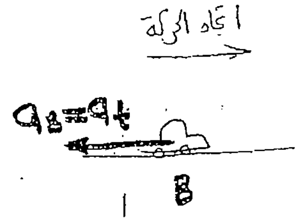
$$\rho_A = \frac{V_A^2}{a_n} \Rightarrow \boxed{\rho_A = 15.55\,m}$$



⇒ **at point B:**

= inflection point ($\rho = \infty$) $\rightarrow$ ($a_n = \text{zero}$)

$$a_B = a_t \Rightarrow \boxed{a_B = -2.41 \ m/s^2}$$



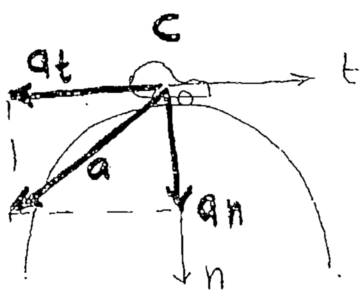
⇒ **at point C:**

$$a_n = \frac{V_c^2}{\rho_c} = \frac{\left(50 \times \frac{5}{18}\right)^2}{150} \Rightarrow \boxed{a_n = 1.286 \ m/s^2}$$

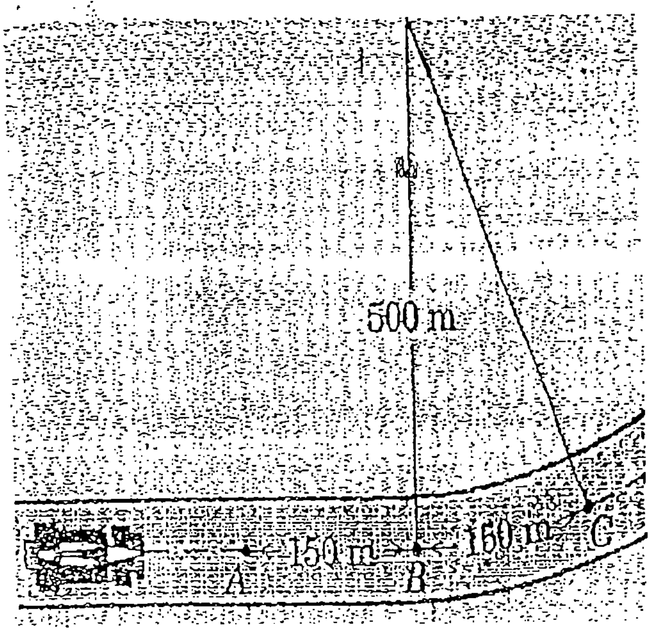
$a_t = -2.41$ (const ثابتة خلال الحركة)

$$a_c = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.41)^2 + (1.286)^2}$$

$$\boxed{a_c = 2.73 \ m/s^2}$$



A car driver is travelling at a speed of 288 km/h on the straightaway. He applied the brakes at point A to reduce the speed at a uniform rate to 216 km/h at point C. Calculate the magnitude of the total acceleration of the car just after passes the point B and at the point C. Draw the acceleration components at point C.



Sol

(a_t = Const)

reduce a speed at a uniform rate.

$$V_C^2 = V_A^2 + 2a_t\,(\Delta S)_{A\to C}$$

$$\left(216 \times \frac{5}{18}\right)^2 = \left(288 \times \frac{5}{18}\right)^2 + 2a_t\,(300)$$

$$\boxed{a_t = -4.67 \quad m/s^2}$$

عجلة تقصيرية
(عكس إتجاه الحركة)

at Point B:

$$V_B^2 = V_A^2 + 2a_t\,(\Delta S)_{A\to B}$$

$$V_B^2 = \left(288 \times \frac{5}{18}\right)^2 + 2(-4.67)(150)$$

$$\boxed{V_B = 70.71 \quad m/s}$$

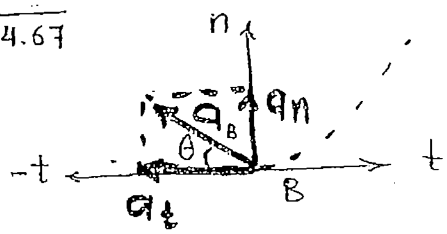
$$a_{n_B} = \frac{V_B^2}{\rho_B} \Rightarrow a_{n_B} = \frac{(70.71)^2}{500} = 10 \quad m/s^2$$

$$a_B = \sqrt{a_t^2 + a_n^2} \Rightarrow a_B = \sqrt{(4.67)^2 + (10)^2}$$

$$\boxed{a_B = 11.037 \ m/s^2}$$

$$\theta = Tan^{-1} \frac{a_n}{a_t} = Tan^{-1} \frac{10}{-4.67}$$

$$\boxed{\theta = -64.96^\circ}$$



at Point C :

اتجاه الحركة

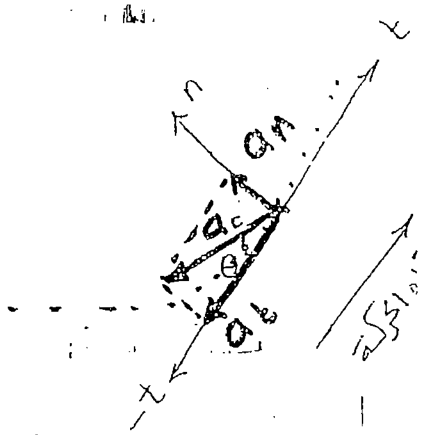
$$a_{n_C} = \frac{V_C^2}{\rho_C} \Rightarrow a_{n_C} = \frac{(216 \times \frac{5}{18})^2}{500} = 7.2 \ m/s^2$$

$$a_C = \sqrt{a_{n_C}^2 + a_t^2} = \sqrt{(4.67)^2 + (7.2)^2}$$

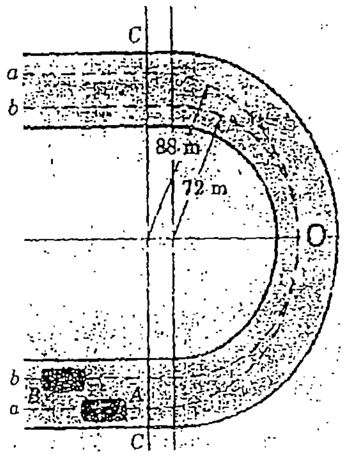
$$\boxed{a_C = 8.58 \ m/s^2}$$

$$\theta = Tan^{-1} \frac{a_n}{a_t} = Tan^{-1} \frac{7.2}{-4.67}$$

$$\boxed{\theta = -57.03^\circ}$$



Race car A follows a circular path a-a while race car B follows another circular one b-b on the marked track. If each car has a maximum speed limited to that corresponding to a lateral (normal) acceleration of 0.8 g, determine the times t_A and t_B for both cars to complete the turn as started and finished by the line C-C. Draw the velocity and acceleration components at the center of the track (point O).

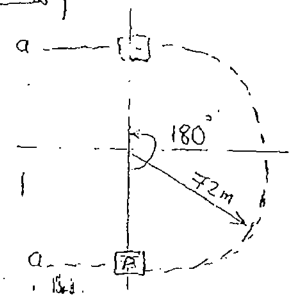


SOL

$V = Const$ (سـرعة السيارتين ثابتة)

$a_t = zero$

$a_{n_A} = a_{n_B} = 0.8g$



For Car A:

$\because V = Const \quad \therefore S_A = V_A \cdot t_A$

$S_A = \text{length of arc} \Rightarrow S_A = 88 \times 180 \times \frac{\pi}{180} = 88\pi.$

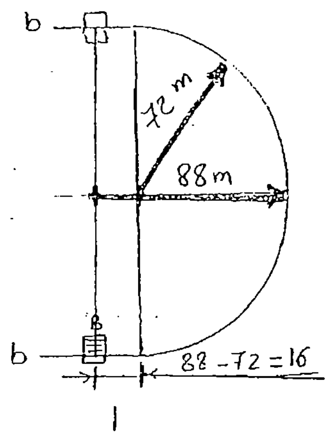
$\boxed{S_A = 88\pi}$

$$a_n = \frac{V_A^2}{\rho_A} \Rightarrow a_n = \frac{V_A^2}{88} \Rightarrow 0.8(9.8) = \frac{V_A^2}{88}$$

$\boxed{V_A = 26.266 \ m/s}$

$$t_A = \frac{S_A}{V_A} = \frac{88\pi}{26.26} = 105 \ sec \quad \boxed{t_A = 105 \ sec}$$

For Car B :



$V = Const$

$S_B = V_B \, t_B$

$$S_B = \underbrace{\left(72 * 180 * \frac{\pi}{180}\right)}_{\text{نصف دائرة}} + \underbrace{16 + 16}_{\text{خط مستقيم}}$$

$$\boxed{S_B = 258.2 \, m}$$

$$a_n = \frac{V_B^2}{\rho_B} \Rightarrow 0.8\,(9.8) = \frac{V_B^2}{72}$$

$$\boxed{V_B = 23.75 \, m/s}$$

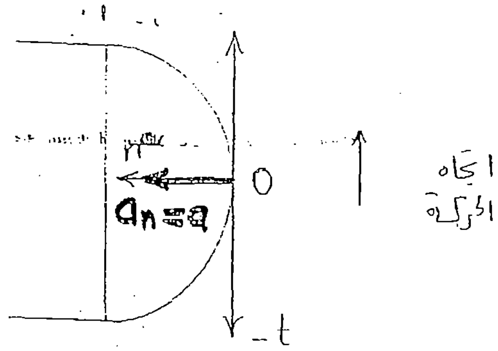
$$t_B = \frac{S_B}{V_B} = \frac{258.2}{23.75}$$

$$\boxed{t_B = 10.87 \ sec}$$

Velocity and acc. Components at point O :

$a_{total} = a_n$

$(a_t = zero)$



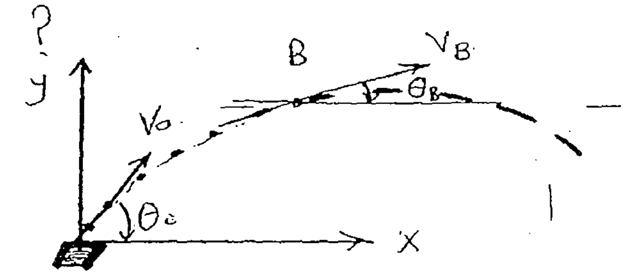
المحاور المشتركة

في هذا الجزء يربط بين النوعين من المحاور ويكون كالآتي :

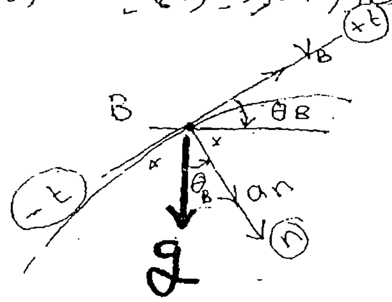
← اذا كان لدينا مسار للمقذوفات (projectile) وطلب radius of curvat. عند نقطة معينة .

Ex: Find ρ at B ?

المطلوب ايجاد نصف القطر عند النقطة B



← لإيجاد نصف القطر هنا يتم وضع المحاور العمودية والمماسية (n, t) عند النقطة (B)



⇐ نحلل العجلة g على محور (n) .

$$a_{n_B} = g \cos\theta_B \leftarrow ①$$

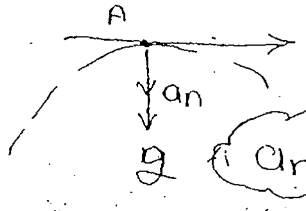
⇐ نعوض في قانون an ⇐ $a_{n_B} = \frac{V_B^2}{\rho_B} \leftarrow ②$

من ① ، ② نستطيع ايجاد قيمة ρ_B

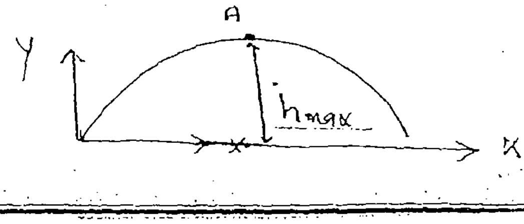
ولكن يجب أولا ايجاد قيم الـ θ_B ، V_B من حركة المقذوفات

$$V_B = \sqrt{V_{B_x}^2 + V_{B_y}^2} \quad , \quad \theta_B = \tan^{-1}\frac{V_{y_B}}{V_{x_B}}$$

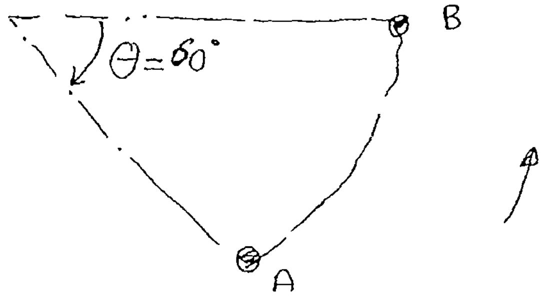
Note : ← اذا طلب الـ ρ عند اقصى نقطة (اعلى ارتفاع) على مسار المقذوف .



$$a_n = g = \frac{V_A^2}{\rho}$$



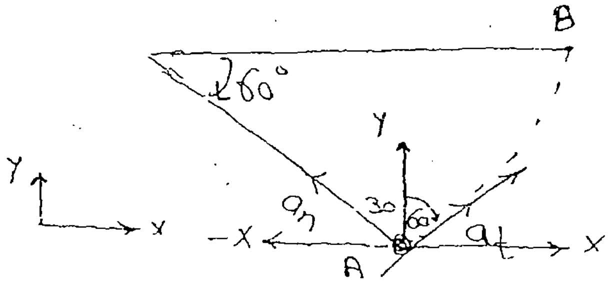
← اذا طلب تحليل مركبات العجلة في اتجاه X واتجاه y :



Req: Find a_x , a_y at Point A and Point B ??

← نرسم a_n ، a_t عند كل نقطة أولاً

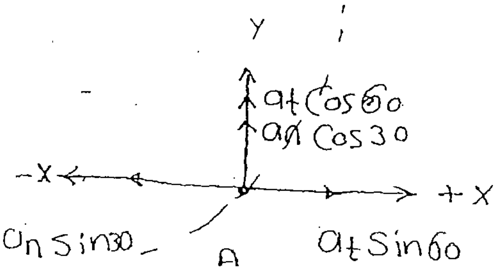
At Point (A) :



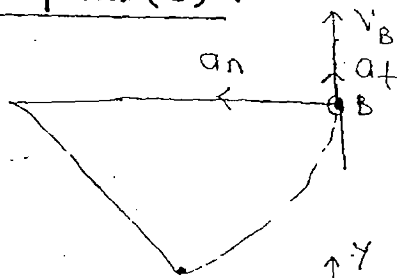
Assume a_t = + value (تزايدية)

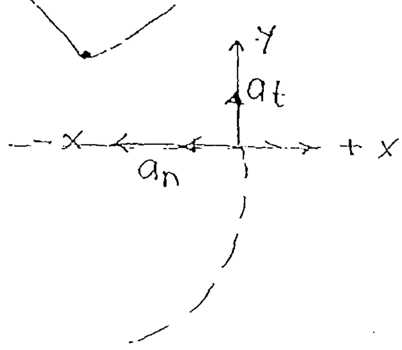
$a_x = a_t \sin 60 - a_n \sin 30$.

$a_y = a_t \cos 60 + a_n \cos 30$.



At Point (B) :

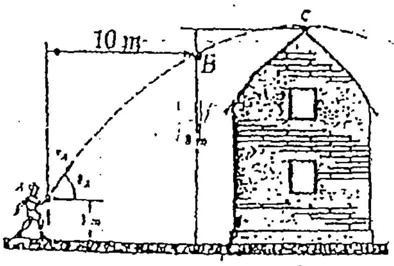




$a_x = -a_n$.

$a_y = +a_t$.

The boy at A attempts to throw a ball over the roof of a country house with an initial speed of $v_A = 20$ m/s. Determine the angle θ_A at which the ball must be thrown so that it just clears the peak at C. Find the radius of curvature of the path at the point B and at the maximum height (point C).

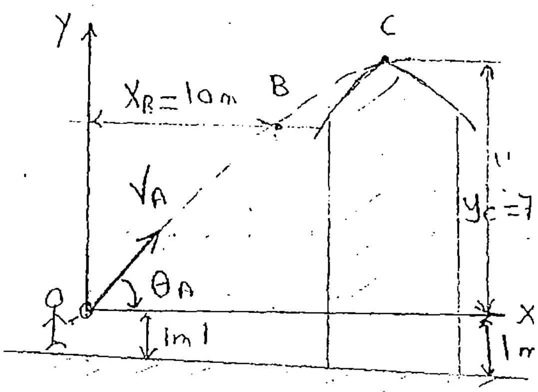


Sol

$V_A = 20\,m/s \qquad \theta_A = ??$

$\rho_B = ?? \quad , \quad \rho_C = ??$

Point C (Peak) $\Rightarrow V_{y_C} = 0$



$\boxed{1}$ to get (θ_A):

نختار نقطة بها معلومات (معلومين) او اكثر .

At Point C: (max hieght)

$V_{y_C} = 0 \quad , \; y_C = 7\,m$

$V_{y_C}^2 = V_{oy}^2 - 2g(y_C - \cancel{y_o})$, $y_o = 0$

$\text{zero} = (20\sin\theta_A)^2 - 2\times9.8(7)$

$\boxed{\theta_A = 35.87^\circ}$

$V_{ox} = V_A \cos\theta_A$

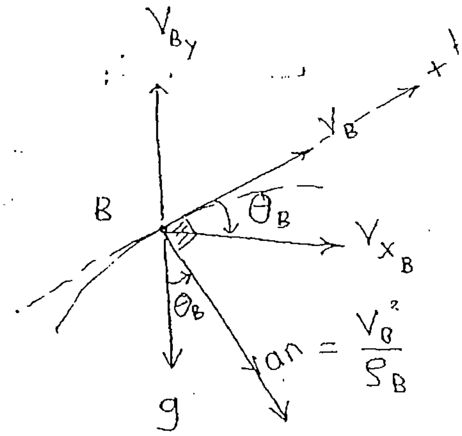
$V_{ox} = 20\cos\theta_A$

$V_{oy} = 20\sin\theta_A$

$\Rightarrow$ to get ρ_B at Point (B):

نحلل ال g ونأخذ مركبتها في إتجاه ال (a_n) .

$a_n = g\cos\theta_B = \dfrac{V_B^2}{\rho_B}$



ولإيجاد ال ρ_B يجب إيجاد

قيم ال V_B ثم θ_B :

لاحظ : تم وضع المحاور (X,Y) و (n,t) معاً

to find (V_B):

$$V_{x_B} = V_{ox} = 20 \cos 35.87^\circ$$

$V_{y_B} \Rightarrow$ يجب ايجاد الـ t_B أولاً

$$X_B = 10m \Rightarrow X_B = V_{ox} . t_B \Rightarrow 10 = 20 \cos 35.87 * t_B$$

$$\boxed{t_B = \frac{10}{16.2} = 0.62\ S}$$

$$V_{y_B} = V_{oy} - g t_B \Rightarrow V_{y_B} = 20 \sin(35.87)(0.62)$$

$$\boxed{V_{y_B} = 5.67\ m/s} \Rightarrow \theta_B = \tan^{-1} \frac{V_{y_B}}{V_{x_B}}$$

$$V_B = \sqrt{V_{x_B}^2 + V_{y_B}^2}$$

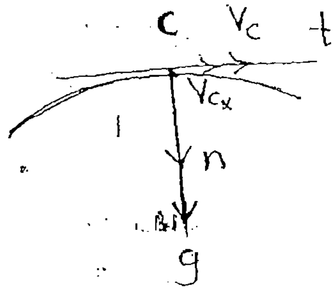
$$V_B^2 = V_{x_B}^2 + V_{y_B}^2 = (16.2)^2 + (5.64)^2 = 294.6\ m/s^2$$

$$V_B = 17.16\ m/s$$

$$g \cos \theta_B = \frac{V_B^2}{\rho_B}$$

$$\boxed{\rho_B = 31.8\ m}$$

At point (c) (max hieght)



$$g = a_n = \frac{V_c^2}{\rho_c}$$

$$\rho_C = \frac{V_c^2}{g}$$

$$V_c = V_{cx} = V_o \cos \theta_A$$

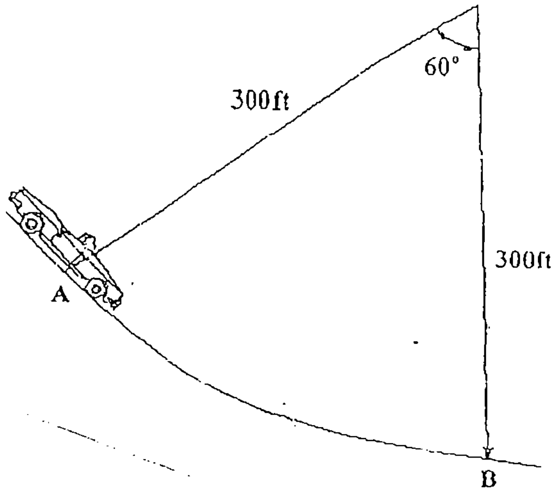
⇐ يجب إيجاد قيمة الـ V_c من معادلات الـ projectile أولاً .

$$\boxed{\rho_c = 26.75m}$$

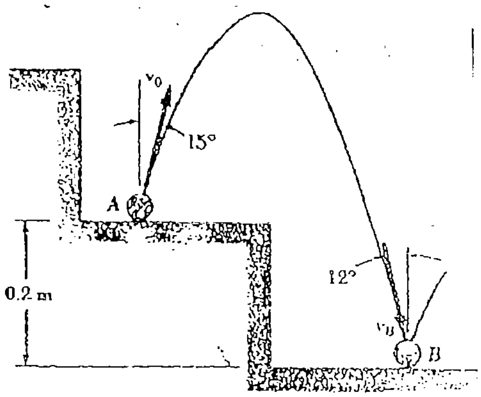
1 A projectile is fired at an angle of 30° above the horizontal with a muzzle velocity of 460 m/s. Find the radius of curvature ρ of its path 10 seconds after firing. Neglect air resistance so that its only acceleration is g down. Also find the rate of change of the magnitude of the velocity.

Question No.: 4:(24 Marks)

2 When the car is at A, it's speed is increased along the vertical circular path at the rate of $v^{\cdot} = 0.3\,t$ ft/s^2, where t in seconds. If it starts from rest at A. determine the magnitudes of its velocity and acceleration when it reaches B.



3 A ball is dropped onto a step at point A and rebounds with a velocity v_o at an angle of $15°$ with the vertical. Determine the value of v_o knowing that just before the ball bounces at point B its velocity v_B forms an angle of $12°$ with the vertical. Also determine the radius of curvature at both A and B.

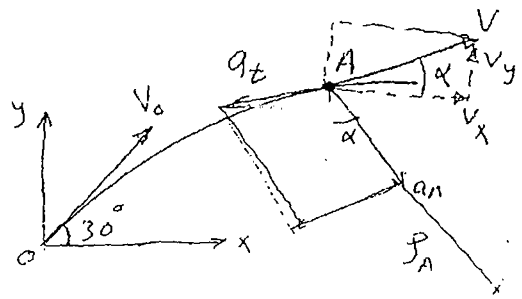


الحل فى الصفحات الآتية

15 Points

Modal Answer

1 $V_o = 460$ m/s $t_A = 10$ s , Find ρ_A and a_t



In x- direction

$V_{x_o} = V_o \cos 30$

$= 460 \times \frac{\sqrt{3}}{2} = 398.37$ m/s

V_x at any time $= V_{x_o} = 398.37$ m/s

In y- direction

$V_{y_o} = V_o \sin 30$

$= 460 \times \frac{1}{2} = \underline{230}$ m/s (4)

At Point A

$V_x = V_{xo} = \underline{398.37}$ m/s

$V_y = V_{y_o} - g t$

$V_y = 230 - 9.81 \times 10 = \underline{131.9}$ m/s (4)

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(398.37)^2 + (131.9)^2} = \underline{419.64}$ m/s

$\alpha = \tan^{-1} \frac{V_y}{V_x} = \tan^{-1} \frac{131.9}{398.37} = \underline{18.32°}$

$a_n = g \cos \alpha = 9.81 \cos 18.325 = \underline{9.31}$ m/s^2 (2)

$a_n = \frac{V^2}{\rho_A}$

$9.31 = \frac{(419.64)^2}{\rho_A} \longrightarrow \rho_A = 18.915 \times 10^3$ m $= \boxed{18.915 \text{ km}}$ (2)

* The rate of change of the magnitude of the velocity is the tangential component of accleration (a_t)

$a_t = -g \sin \alpha = -9.81 \sin 18.325 \boxed{-3.08 \text{ m/s}^2}$ (3)

Best wishes

Dr. El-Adl

Apr. 2010

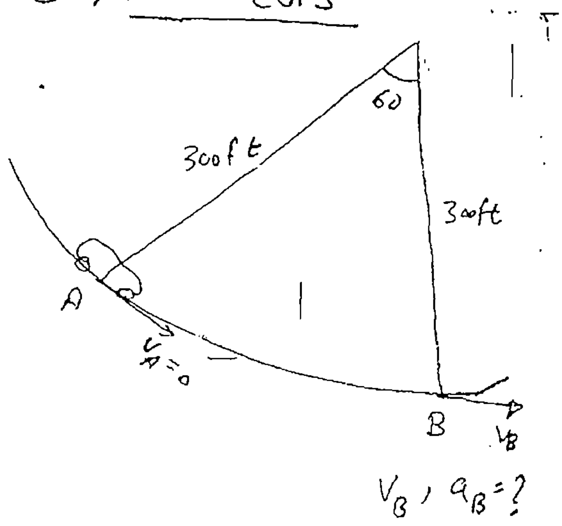
Prob (9) , 2012 – 2013

(24)

2 $a_t = 0.3\,t \quad ft/s^2$

at point A $\quad V_A = 0 \,, \; a_t = 0$

Find V and a at point B



$$a_t = \frac{dv}{dt} = 0.3\,t$$

$$\int_{V_A}^{V_B} dv = \int_0^t 0.3\,t \; dt$$

$$V_B = 0.\frac{3}{2}\,t^2 = 0.15\,t^2 = \frac{ds}{dt} \quad (8) \qquad s = r\theta$$

$$\int_0^s ds = \int_0^t 0.15\,t^2\,dt$$

$$s = \frac{0.15}{3}\,t^3 = 0.05\,t^3 \quad (8)$$

at point B $\longrightarrow$ $S_B = 300 \times \frac{60 \times \pi}{180} = 100\,\pi \;\; ft \quad (4)$

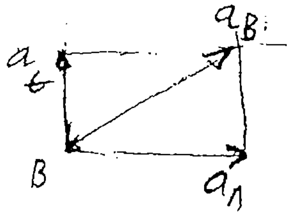
$$100\,\pi = 0.05\,t^3 \longrightarrow t = \sqrt[3]{\frac{100\,\pi}{0.05}} = \sqrt[3]{6283} = 18.45\,s$$

$$V_B = 0.15 \times (18.45)^2 = 51 \quad ft/s$$

$$a_B)_t = 0.3 \times (18.45) = 5.54 \quad ft/s^2$$

$$a_B)_n = \frac{V_B^2}{\rho} = \frac{(51)^2}{300} = 8.67 \quad ft/s^2 \quad (5)$$

$$a_B = \sqrt{(5.54)^2 + (8.67)^2} \simeq 10.29 \quad ft/s^2$$



3

$V_{xo} = V_o \sin 15^\circ$ $\quad V_{yo} = V_o \cos 15^\circ$

$V_{xB} = V_B \sin 12^\circ$ $\quad V_{yB} = V_B \cos 12^\circ$ $\quad y_B = -0.2\,m$

Since the horizontal velocity is constant,

$V_{xo} = V_{xB}$ then;

$V_o \sin 15 = V_B \sin 12$

$\frac{V_o}{V_B} = \frac{\sin 12}{\sin 15} \longrightarrow V_o = 0.8\, V_B$ —— (1)

In y-direction

$V_{yB}^2 = V_{yo}^2 - 2g\, y_B$

$V_B^2 \cos^2 12 = V_o^2 \cos^2 15 - 2 \times 9.81 \times (-0.2)$

$0.95677 V_B^2 = 0.933\, V_o^2 + 3.924$ —— (2)

substituting (1) in (2)

$V_B^2 (0.9568 - 0.602) = 3.924 \longrightarrow V_B = \pm 3.324\ m/s$

$V_B = -3.324\ m/s$

$V_o = 0.8\, V_B = 2.67\ m/s$

$V_B = V_o - g\, t_B$

$-3.324 = 2.67 - 9.8\, t_B$

$t_B = 0.61\ s$

$a_n = g \cdot \sin 15^\circ = \frac{V^2}{\rho} = 9.81 \sin 15^\circ$

$\rho_o = V_o^2 / 9.81 \sin 15^\circ = \frac{(2.67)^2}{9.81 \sin 15^\circ} = 2.8\ m$

$\rho_B = V_B^2 / 9.81 \sin 12^\circ = \frac{(3.324)^2}{9.81 \sin 12^\circ} = 5.42\ m$